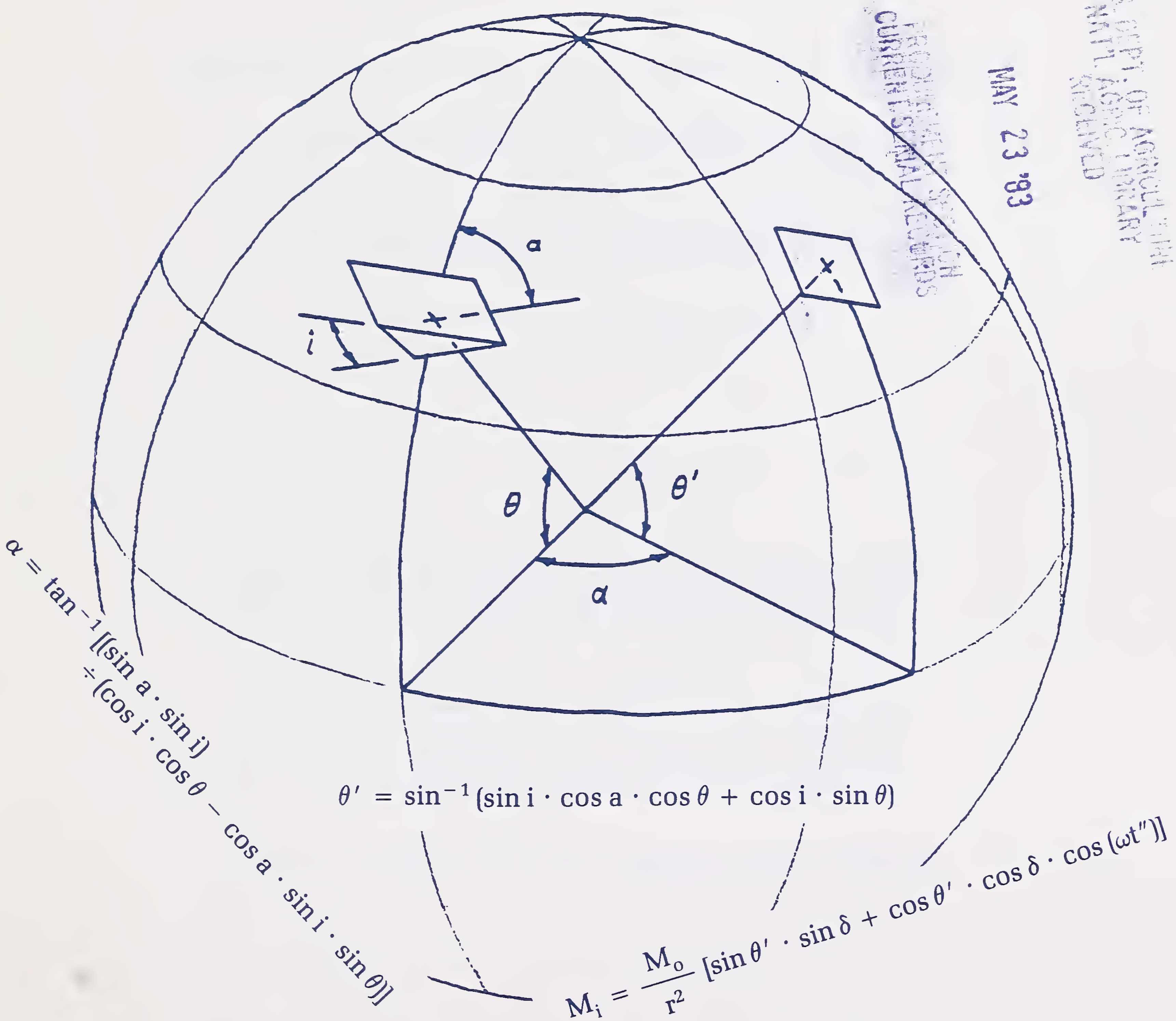
Determination of Potential Direct Beam Solar Irradiance

Merrill R. Kaufmann and James D. Weatherred



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Abstract

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Keywords: Solar irradiance, direct beam irradiance, latitude, declination, azimuth, inclination

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Determination of Potential Direct Beam Solar Irradiance

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Introduction

Potential direct beam solar irradiance is the radiation received above the earth's atmosphere. The maximum direct beam irradiance is received when a surface above the atmosphere is perpendicular to the sun's rays. For incoming shortwave irradiance, this is equivalent to the solar constant, generally taken to be 1360 $W \cdot m^{-2}$ (1.95 $cal \cdot cm^{-2} \cdot min^{-1}$).

Actual irradiance received at the earth's surface is lower than the potential direct beam irradiance. The reduction results from a number of factors, some of which are highly predictable because they are geometric in nature, and some of which are less predictable because of varying atmospheric effects.

Geometric effects include latitude, solar declination, distance between the sun and earth (radius vector), inclination and aspect of the plane in question, time of day, and obstructions. The plane or surface may be a hillside, leaf, wall or roof of a building, solar collector, etc. Atmospheric effects include depth of clear atmosphere through which radiation must pass (a function of solar declination, latitude, elevation, and time of day), radiation scattering (a function of atmospheric composition and contamination), and cloudiness (Gates 1980).

While knowledge of the actual irradiance received at a surface is useful to hydrologists, plant physiologists, solar energy engineers, and others, the complexities of predicting actual irradiance are great because atmospheric conditions are dynamic. Knowledge of potential direct beam irradiance is useful because potential irradiance is a maximum limit for actual irradiance and is usually used to derive estimates of actual irradiance.

Incoming shortwave irradiance is of interest to hydrologists, micrometeorologists, and solar engineers because of its direct role in determining the energy balance of a given site. In some cases, however, only a portion of the solar spectrum is of concern. For example, a plant physiologist studying photosynthesis or stomatal behavior may be concerned only with photosynthetic photon flux density (visible irradiance, 400 to 700 nm), while a human pathologist evaluating skin disorders may be interested in the ultraviolet portion of the spectrum.

Frank and Lee (1966) provided tables giving the times of sunrise and sunset and the total daily incoming irradiance, which covered a range of latitudes, aspects, slopes, and times of year. Buffo et al. (1972) also provided extensive tables and figures of direct beam solar irradiance. Swift (1976) presented an algorithm for calculating daily total solar irradiance on mountain slopes. This algorithm is suitable for use as a computer subroutine, making the use of tables unnecessary. Formulas for calculating direct beam irradiance presented by Frank and Lee (1966), Gates (1980), and Lee (1978) are useful, but they require more extensive development by the user for routine prediction of irradiance. Furthermore, the formulas are considered mainly in the context of total incoming shortwave irradiance.

This paper has two objectives. First, the formulas used by Frank and Lee (1966), Buffo et al. (1972), and Swift (1976) are extended to provide more details of the calculation procedures, particularly with regard to determinations of hour angles for calculating sunrise, sunset, and radiation and of the effects of intervening obstruction by terrain. This is done to facilitate the use of computers for calculations of both instantaneous and total direct beam irradiance. The reader is referred to the programs listed in the appendixes and to Swift's (1976) algorithm for linking the calculations.[3]

Second, the formulas are presented as a general case for direct beam solar irradiance, where the incoming irradiance may be total shortwave, visible, ultraviolet, etc., as selected by the user. In effect, the formulas for instantaneous and total irradiance yield a "multiplication factor" to be taken times the potential irradiance (e.g., the solar constant for total solar shortwave irradiance, approximately 2,600 $\mu E \cdot m^{-2} \cdot sec^{-1}$ for 400 to 700 nm irradiance, etc.). Duffie and Beckman (1974) provide data from several sources describing the standard spectral distribution of extraterrestrial irradiance. It must be recognized that the calculation procedures given below pertain only to the determination of irradiance above the atmosphere. Atmospheric effects on actual irradiance received at a surface in question must be determined for the appropriate portion of the solar spectrum of interest.

Abbreviations

- M_o — potential irradiance constant (given a dimensionless value of 1.0 for a surface above the atmosphere, perpendicular to the sun's rays, at the equinox)
- R_o — potential irradiance constant in user's units
- M_i — multiplier for instantaneous irradiance for a given latitude, aspect, inclination, time of year, and time of day
- R_i — instantaneous irradiance in user's units
- M_t — multiplier for total daily irradiance from sunrise to sunset for a given latitude, aspect, inclination, and time of year

[3]*For those who have access to a Hewlett Packard 41C hand calculator, a Users' Library Solutions Manual for Solar Engineering (No. 00041-90138) has a program for solar-beam irradiation.*

R_t — total daily irradiance in user's units
r — radius vector, the ratio of the distance between the sun and earth at a given date to the mean distance
n — Julian date
m — month of year (1 to 12)
d — day of month
θ — latitude in degrees of the given slope, positive in northern hemisphere, negative in southern hemisphere
θ' — latitude of the equivalent slope
δ — solar declination in degrees for the given slope
α — change in hour angle in degrees from the given to the equivalent slope
ω — angular velocity of the earth's rotation ($15° \cdot hr^{-1}$)
t — time in hours from solar noon
- t_1 — sunrise at the given slope
- t_2 — sunset at the given slope
- t_1' — sunrise at the equivalent slope
- t_2' — sunset at the equivalent slope
- t'' — time at the equivalent slope for calculating instantaneous irradiance

a — azimuth of the slope in degrees measured clockwise from north
i — inclination of the slope, 0° to 90°
A — solar altitude in degrees
A' — solar azimuth in degrees measured clockwise from south

Equations

Horizontal Surfaces

The basic equation for calculating instantaneous irradiance on a horizontal surface outside the atmosphere is given by Frank and Lee (1966):

$$R_i = \frac{R_o}{r^2} (\sin\theta \cdot \sin\delta + \cos\theta \cdot \cos\delta \cdot \cos\omega t). \quad [1]$$

The value of r^2 ranges from 0.96676 on January 3 to 1.03370 on July 5 (List 1971, table 169). As an approximation, r^2 can be estimated by first determining the Julian date and declination as follows:

$$n = 31(m-1) + d - 0.4\,m - 1.8. \quad [2]$$

Round n to the nearest day. If m is 1 (January), add 2 to n. If m is 2 (February), add 3 to n. In leap years, add 1 day if m is 3 or greater (Ball 1978).

Solar declination can be estimated:

$$\delta = 23.5 \cdot \sin[0.9863(284 + n)]. \quad [3]$$

Finally, r^2 is calculated:

$$r^2 = 0.999847 + 0.001406(\delta). \quad [4]$$

By these procedures, δ is estimated within $+0.73°$ to $-1.38°$, and r^2 is estimated within +0.84% to −0.70%, depending upon time of year.

Multipliers for instantaneous irradiance may be calculated with a formula similar to equation 1:

$$M_i = \frac{M_o}{r^2} (\sin\theta \cdot \sin\delta + \cos\theta \cdot \cos\delta \cdot \cos\omega t) \quad [5]$$

where M_o is assigned a value of 1.0 at the equinox. In later illustrations, values of M_i are plotted as a function of several variables. The user may calculate instantaneous irradiance using a selected R_o as follows:

$$R_i = M_i\,R_o. \quad [6]$$

For a horizontal surface, if the sum of the absolute values of θ and δ is less than or equal to 90°, the hour angles of sunrise ($-\omega t$) and sunset (ωt) before and after solar noon can be found as follows:

$$\omega t = \cos^{-1}(-\tan\theta \cdot \tan\delta). \quad [7]$$

However, if the sum of the absolute values of θ and δ is greater than 90°, then the sun remains either above or below the horizon all day. If θ and δ have the same sign, the sun remains above the horizon and ωt equals 180°. If θ and δ have different signs, the sun is below the horizon for the day and $\omega t = 0°$.

Times for sunrise (t_1) and sunset (t_2) in hours before or after mean true solar noon are given by:

$$t_2 = -t_1 = \omega t/15. \quad [8]$$

The solar times calculated above are mean solar times. True solar time differs from mean solar time by an amount that varies through the year. True solar time may be calculated by algebraically adding a correction, called the equation of time, which ranges from −14 to +16 minutes (List 1971, table 169). Frank and Lee (1966) give procedures for converting solar time to local standard time.

Multipliers for total irradiance for a solar day may be calculated by integrating equation [5]:

$$M_t = (M_o/r^2)\,[2t \cdot \sin\theta \cdot \sin\delta + (12/\pi) \cdot \cos\theta \cdot \cos\delta \cdot 2 \cdot \sin\omega t]. \quad [9]$$

The user may calculate total irradiance using a selected R_o as follows:

$$R_t = M_t \cdot R_o. \quad [10]$$

Note that M_t has time units of hours; therefore R_o should have units of hours.

An example set of calculations for a horizontal surface is given in the Appendix (example 1).

Tilted Surfaces

When a surface is not horizontal, an "equivalent" horizontal surface exists at a higher or lower latitude and longitude which is parallel to the given surface. This equivalent surface is used to calculate the instantaneous and total irradiance received on the given surface. The next series of formulas, extending those of Frank and Lee (1966, citing earlier references), determine the location of the equivalent surface. Refer to figure 1 for a depiction of a given and equivalent slope.

The latitude of the equivalent surface is determined as follows:

$$\theta' = \sin^{-1}(\sin i \cdot \cos a \cdot \cos\theta + \cos i \cdot \sin\theta) \qquad [11]$$

where inclination is in degrees and azimuth is 0° to 360°. If inclination is given in percent slope, then:

$$i = \tan^{-1}(\%\ \text{slope}/100). \qquad [12]$$

The longitudinal correction of hour angle for the equivalent surface is given by:

$$\alpha = \tan^{-1}[(\sin a \cdot \sin i) \div (\cos i \cdot \cos\theta - \cos a \cdot \sin i \cdot \sin\theta)] \qquad [13]$$

when a is greater than 0°.

Because the $\tan^{-1}$ of an angle is defined for angles between $-90°$ and 90°, if the slope is east-facing and α is less than 0°, then α is added to 180°. If the slope is west-facing and α is greater than 0°, α is added to $-180°$. If a is 0° (due north) and the sum of $|\theta|$ and i is less than or equal to 90°, α equals 0°. However, if the sum of $|\theta|$ and i is greater than 90°, α equals 180°.

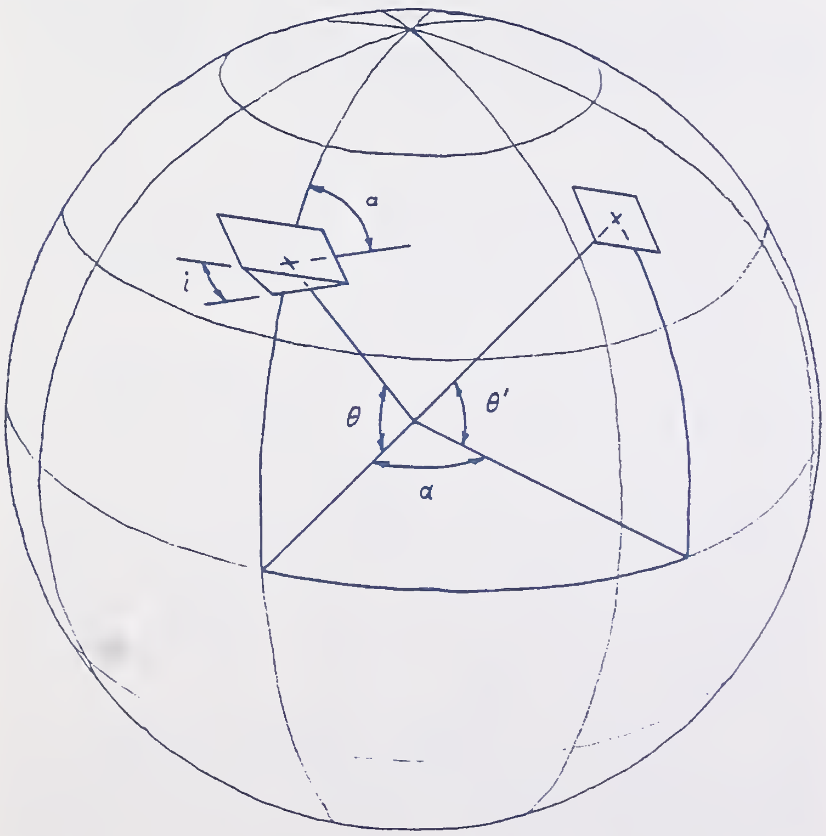

Figure 1.—Relation of given slope and equivalent slope. Given slope at latitude θ has azimuth a and inclination i. Equivalent slope has a different latitude, θ', and the longitude differs from that of the given slope by the hour angle, α.

Hour angles for sunrise and sunset at the equivalent surface are calculated the same way as those for horizontal slopes in equation [7], except that the equivalent latitude is used:

$$\omega t' = \cos^{-1}(-\tan\theta' \cdot \tan\delta). \qquad [14]$$

The same limits apply to $(-\tan\theta' \cdot \tan\delta)$ and $\omega t'$ using θ' as to $(-\tan\theta \cdot \tan\delta)$ and ωt for equation [7]. The special case of a double sunrise and sunset is considered later.

Hour angles for sunrise and sunset of the given slope may be determined as follows. If $(-\omega t + \alpha)$ is greater than or equal to $\omega t'$ (values from eq. [7], [13], and [14]), then the given slope is completely shaded throughout the day (t_1 and t_2 equal 0). If the slope is not completely shaded, the time of sunrise at the given slope then becomes:

$$t_1 = [\text{maximum of } (-\omega t) \text{ and } (-\omega t' - \alpha)] / 15. \qquad [15]$$

The time of sunset at the given slope becomes:

$$t_2 = [\text{minimum of } (\omega t) \text{ and } (\omega t' - \alpha)] / 15. \qquad [16]$$

Irradiance calculations for the equivalent surface require sunrise and sunset times for the equivalent surface. Sunrise for the equivalent slope is:

$$t_1' = [\text{maximum of } (-\omega t + \alpha) \text{ and } (-\omega t')] / 15. \qquad [17]$$

Sunset for the equivalent slope is:

$$t_2' = [\text{minimum of } (\omega t + \alpha) \text{ and } (\omega t')] / 15. \qquad [18]$$

Finally, irradiance multipliers can be calculated. Instantaneous irradiance is determined as follows:

$$M_i = \frac{M_o}{r^2}[\sin\theta' \cdot \sin\delta + \cos\theta' \cdot \cos\delta \cdot \cos(\omega t'')] \qquad [19]$$

where $\omega t'' = \omega t + \alpha$ and t is greater than or equal to t_1 and less than or equal to t_2. If t is less than t_1 or greater than t_2, then M_i equals 0.

A series of graphs discussed below present M_i as a function of latitude, azimuth, inclination, date, and time of day. Recalling equation [6], instantaneous irradiance in user units (R_i) can be determined as the product of M_i and R_o. Alternatively, the numerator value of M_o in equation [19] may be replaced with R_o.

Multipliers for total daily irradiance are calculated:

$$M_t = (M_o/r^2)\,[(t_2' - t_1') \cdot \sin\theta' \cdot \sin\delta + (12/\pi) \cdot \cos\theta' \cdot \cos\delta \cdot (\sin\omega t_2' - \sin\omega t_1')]. \qquad [20]$$

Total irradiance in user units (R_t) is determined as the product of M_t and R_o, or R_t may be determined by

substituting R_o for the numerator value of M_o in equation [20]. Values of R_t, using the solar constant as R_o, for a range of latitudes, azimuths, inclinations, and dates are tabulated in Frank and Lee (1966).

An example set of computations for a tilted surface is given in the Appendix (example 2).

Double Sunrise and Sunset

Steep, north-facing slopes and the north sides of buildings at middle latitudes in the northern hemisphere exhibit a sunrise and sunset both in the morning and in the afternoon. Double sunrises and sunsets occur when the following conditions are met:

$$(-\omega t') < (\omega t + \alpha - 360) < (-\omega t + \alpha) < (\omega t'). \qquad [21]$$

If these conditions exist, sunrise and sunset times for the given slope are calculated as follows:

$$\text{First } t_1 = -\omega t / 15 \qquad [22]$$
$$\text{First } t_2 = (\omega t' - \alpha) / 15 \qquad [23]$$
$$\text{Second } t_1 = (360 - \omega t' - \alpha) / 15 \qquad [24]$$
$$\text{Second } t_2 = \omega t / 15. \qquad [25]$$

For calculating irradiance at the equivalent surface, sunrise and sunset times are:

$$\text{First } t_1' = -\omega t' / 15 \qquad [26]$$
$$\text{First } t_2' = (\omega t + \alpha - 360) / 15 \qquad [27]$$
$$\text{Second } t_1' = (-\omega t + \alpha) / 15 \qquad [28]$$
$$\text{Second } t_2' = \omega t' / 15. \qquad [29]$$

Calculation of the total daily irradiance multiplier for days having two sunrises and sunsets requires determining M_t using equation [20] for the first solar day (first t_1' to t_2') and for the second solar day (second t_1' to t_2'). These values are added to obtain the total daily irradiance multiplier. Again, total daily irradiance in user's units may be determined as the product of M_t and R_o or by replacing the numerator value of M_o with R_o in equation [20].

An example set of calculations for double sunrise and double sunset times is given in the Appendix (example 3). For radiation calculations on days of double sunrise and sunset, refer to example 2.

Obstruction of the Horizon

In mountainous regions, few sites are characterized by a direct view of the earth's horizon. Unless sites are located upon the highest ridgetops, intervening hills and ridges obscure the sun, resulting in later sunrises or earlier sunsets than those predicted by equation [8]. The delay of sunrise, advancement of sunset, or midday obstruction by intervening terrain (or objects) can be determined by comparing the elevation angle of the obstruction with the solar altitude. When the solar altitude at a given azimuth is below the obstruction altitude at the same azimuth, irradiance is set to zero.

Procedures for determining the solar altitude and azimuth are given by Lee (1978). The solar altitude is calculated as follows:

$$A = \sin^{-1}(\sin\theta \cdot \sin\delta + \cos\theta \cdot \cos\delta \cdot \cos\omega t). \qquad [30]$$

Values of A less than 0° indicate that the sun is below the observer's true horizon, while values greater than 0° indicate that the sun is above the observer's true horizon.

The computation of solar azimuth is somewhat more difficult. Solar azimuths calculated below are determined clockwise from the observer's south. If $|\theta|$ is greater than $|\delta|$, the solar azimuth is given by

$$A' = \sin^{-1}(\cos\delta \cdot \sin\omega t / \cos A). \qquad [31]$$

To establish the correct quadrant, first calculate the hour angle (H_w) at which the sun is due west of the observer:

$$H_w = \cos^{-1}(\tan\delta \cdot \cot\theta). \qquad [32]$$

Then, if $|\omega t|$ is less than or equal to H_w, A' is in the correct quadrant. However, if $|\omega t|$ is greater than H_w, A' is placed in the correct quadrant by

$$A' = \operatorname{sign}(180, \omega t) - A' \qquad [33]$$

where sign (180, ωt) is 180° if ωt is greater than or equal to 0 and −180° if ωt is less than 0°.

If $|\theta|$ is less than or equal to $|\delta|$, three possible cases exist. In the first case, $|\omega t|$ is greater than 0° but less than 180°. If δ equals 0°, then θ equals 0° and

$$A' = \operatorname{sign}(90, \omega t). \qquad [34]$$

If δ is not equal to 0°, then A' is calculated as in equation [31]. A' from equation [31] is in the correct quadrant if δ is less than 0°. For δ greater than 0°, A' is determined using equation [33].

In the second case, ωt equals 0°. If $|\theta|$ equals $|\delta|$, then when θ is greater than or equal to 0°, A' is 0°; when θ is less than 0°, A' is 180°. If $|\theta|$ is less than $|\delta|$, then when δ is greater than 0°, A' is 180°; when δ is less than 0°, A' is 0°.

In the third case, $|\omega t|$ equals 180°. If $|\theta|$ equals $|\delta|$, then when θ is greater than or equal to 0°, A' is 180°, and when θ is less than 0°, A' is 0°. If $|\theta|$ is less than $|\delta|$, then when δ is greater than 0°, A' is 180°, and when δ is less than 0°, A' is 0°.

To determine the effects of intervening terrain, the terrain altitude and azimuth must be determined for the opposite horizon (east horizon for a slope having an azimuth of 0° to 180° and west horizon for a slope having an azimuth of 180° to 360°). This may be done by mapping the elevation angle of the horizon at suitably spaced azimuths or by calculating the elevation angle using topography maps.

At some sites, such as lower, south-facing slopes in valleys (northern hemisphere), the entire horizon between the points of sunrise and sunset must be mapped to determine if the sun is obstructed by the opposite ridge. The portion of the obstructed horizon which must be mapped and the azimuth difference between points at which elevation angle is measured will depend upon the particular site in question and the accuracy required.

After the terrain profile is determined, solar altitude is compared with the terrain elevation at various solar azimuths to determine the times at which the obstruction occludes the sun. For instantaneous irradiance calculations, M_i or R_i is set to zero if the solar altitude is less than the elevation angle of the terrain. For total daily irradiance, M_t or R_t are calculated using t_1' and t_2' adjusted for the effects of slope at the given site and for the effects of terrain elevation.

Explanation of Irradiance Figures

A series of graphs was prepared to illustrate how the instantaneous irradiance multiplier is influenced by latitude, date, azimuth (aspect), inclination (percent slope), and time of day. Three figures are presented for each of three latitudes—30°, 40°, and 50° N. These figures are for June 22, March or September 22, and December 22, representing the summer solstice, vernal and autumnal equinoxes, and winter solstice in the northern hemisphere. Slopes in degrees are as follows: 0% (0°), 20% (11.3°), 50% (26.6°), 100% (45°), 200% (63.4°), 400% (76.0°). Effects of obstructed opposite horizons are not taken into account.

The figures illustrate the complex geometric relationships among latitude, solar declination, azimuth, inclination, and hour angle. Total day length decreases from June 22 to December 22, but the change in day length is greatest at high latitudes. For example, at 30° N latitude, day length decreases from 13.9 hours to 10.1 hours (figs. 2 and 4), while at 50° N latitude, day length decreases from 16.2 hours to 7.8 hours (figs. 8 and 10).

Vertical dashed lines in the figures represent sunrises or sunsets at the true horizon for surfaces tilted east or west. For east-facing surfaces (azimuth of 90°), an increase in slope increases the intensity of radiation immediately after the sun rises past the horizon. However, steeper slopes also experience earlier sunsets because the effective horizon is elevated. Opposite effects occur on west-facing slopes.

Instantaneous irradiance of north-facing slopes (azimuth of 0°) at solar noon decreases as the inclination increases. At steep inclinations, the sun may set in the morning and rise in the afternoon during summer months; this effect becomes more pronounced at higher latitudes (compare figs. 2, 5, 8). During winter months, low sun angles prevent any direct beam radiation from reaching steep, north-facing slopes (figs. 4, 7, 10). On south-facing slopes (azimuth of 180°), instantaneous irradiance is often higher than on horizontal surfaces, because the normal to the slope is close to the solar beam pathway.

Figures 2 through 10 are presented only to demonstrate the effects of various factors which influence instantaneous irradiance. Through proper use of the equations given above, the user may calculate instantaneous irradiance for any latitude, date, azimuth, or inclination. In the southern hemisphere, instantaneous irradiance multipliers for slopes facing directly east or west are similar to those shown in the figures. However, for north- and south-facing slopes, the graphs are reversed.

The total irradiance multiplier (M_t), calculated with appropriate equations above, is represented by the area under the curves between sunrise and sunset. Frank and Lee (1966) tabulated total daily irradiance for latitudes between 30° and 50° N, 16 azimuths, and slopes from 0% to 100% for various times of the year. Their values were calculated with a solar irradiance constant of $2.0\ \mathrm{cal} \cdot \mathrm{cm}^{-2} \cdot \mathrm{min}^{-1}$. Irradiance multipliers for total solar irradiance (M_t) are equal to Frank and Lee's values divided by 120 (from $2\ \mathrm{cal} \cdot \mathrm{cm}^{-2} \cdot \mathrm{min}^{-1}$ times 60 min). Tabulated values of Buffo et al. (1972) include an atmospheric transmission coefficient and cannot be compared directly with those given here or in Frank and Lee (1966).

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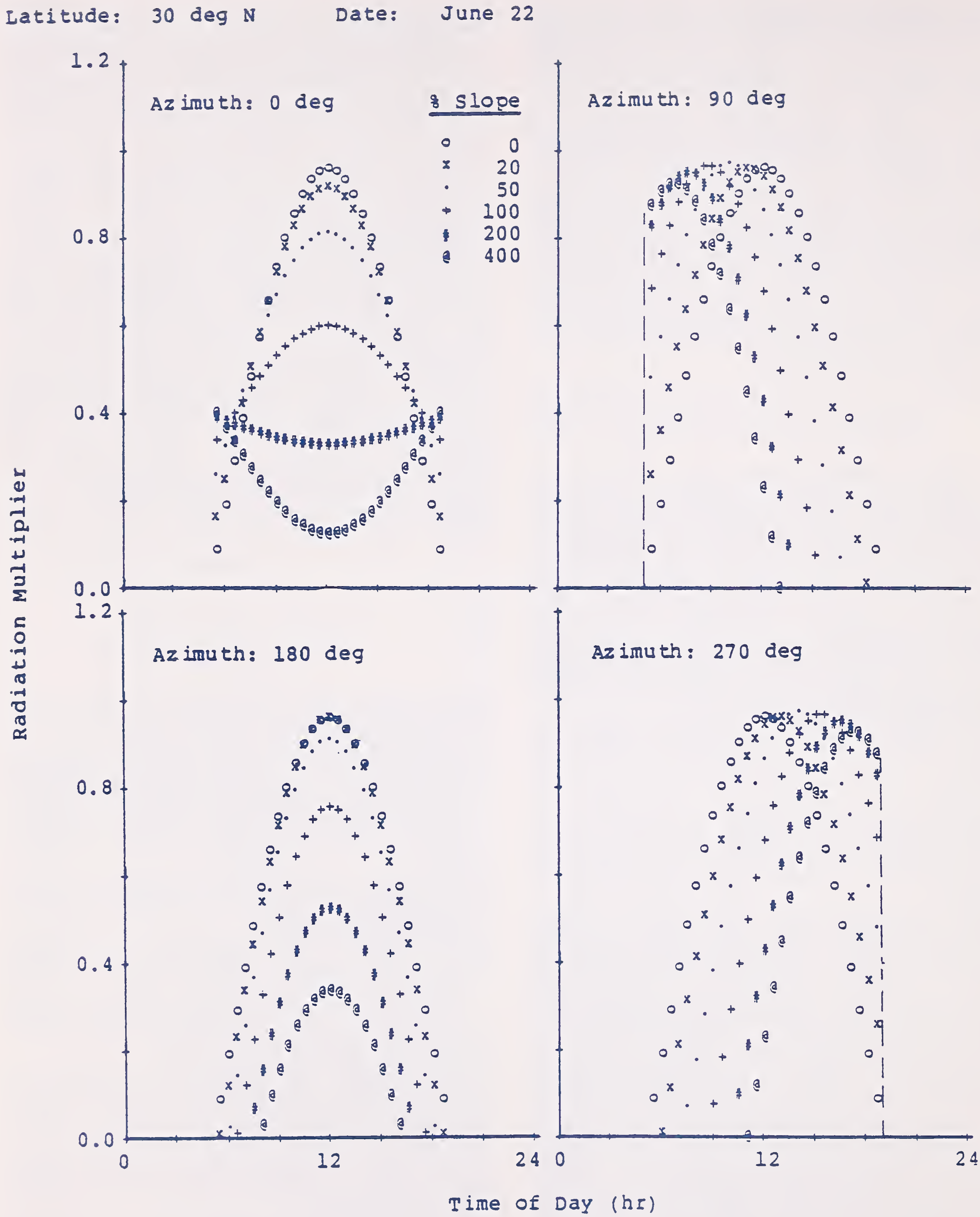

Figure 2.—Instantaneous irradiance multiplier for 30° N latitude on June 22.

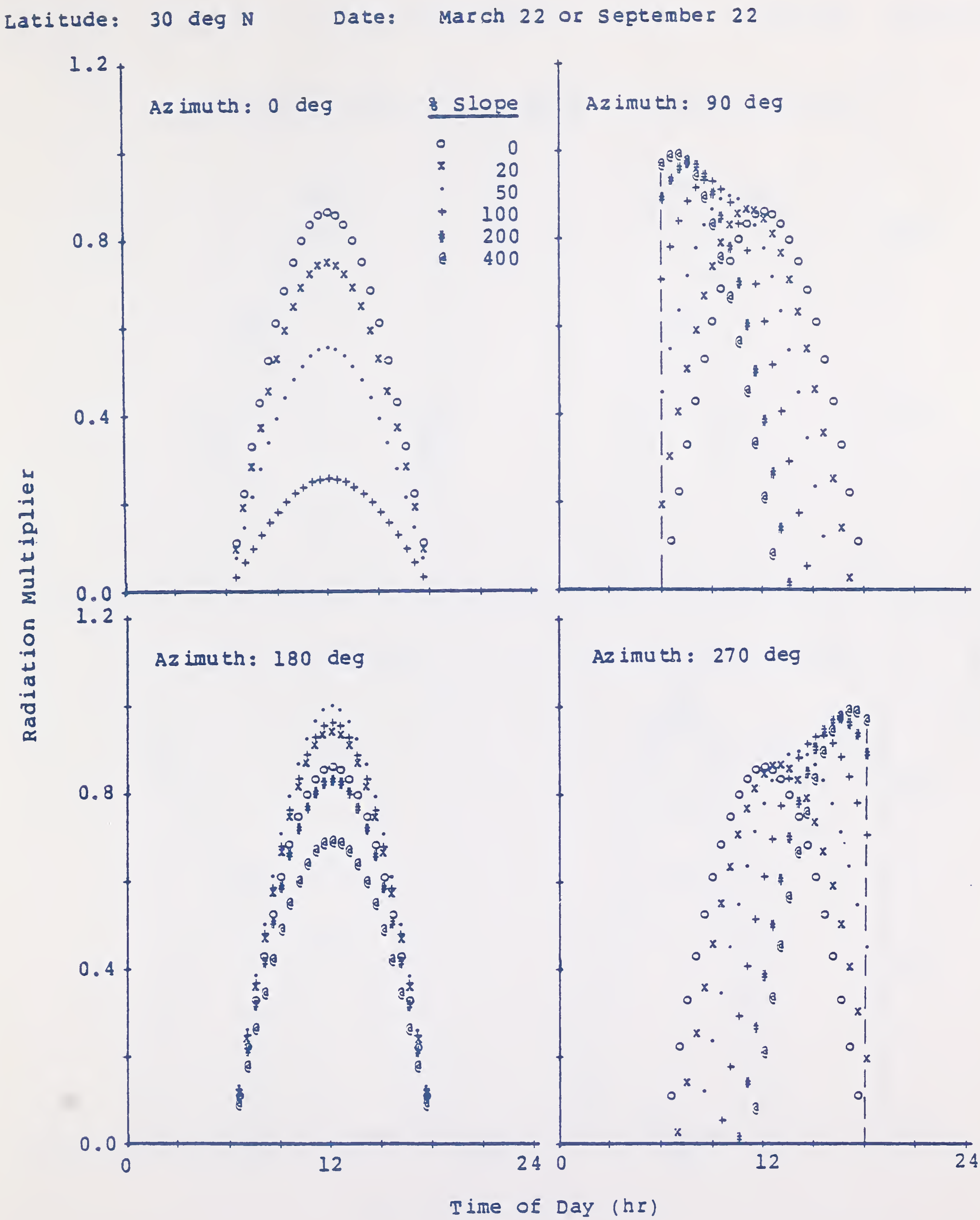

Figure 3.—Instantaneous irradiance multiplier for 30° N latitude on March 22 or September 22.

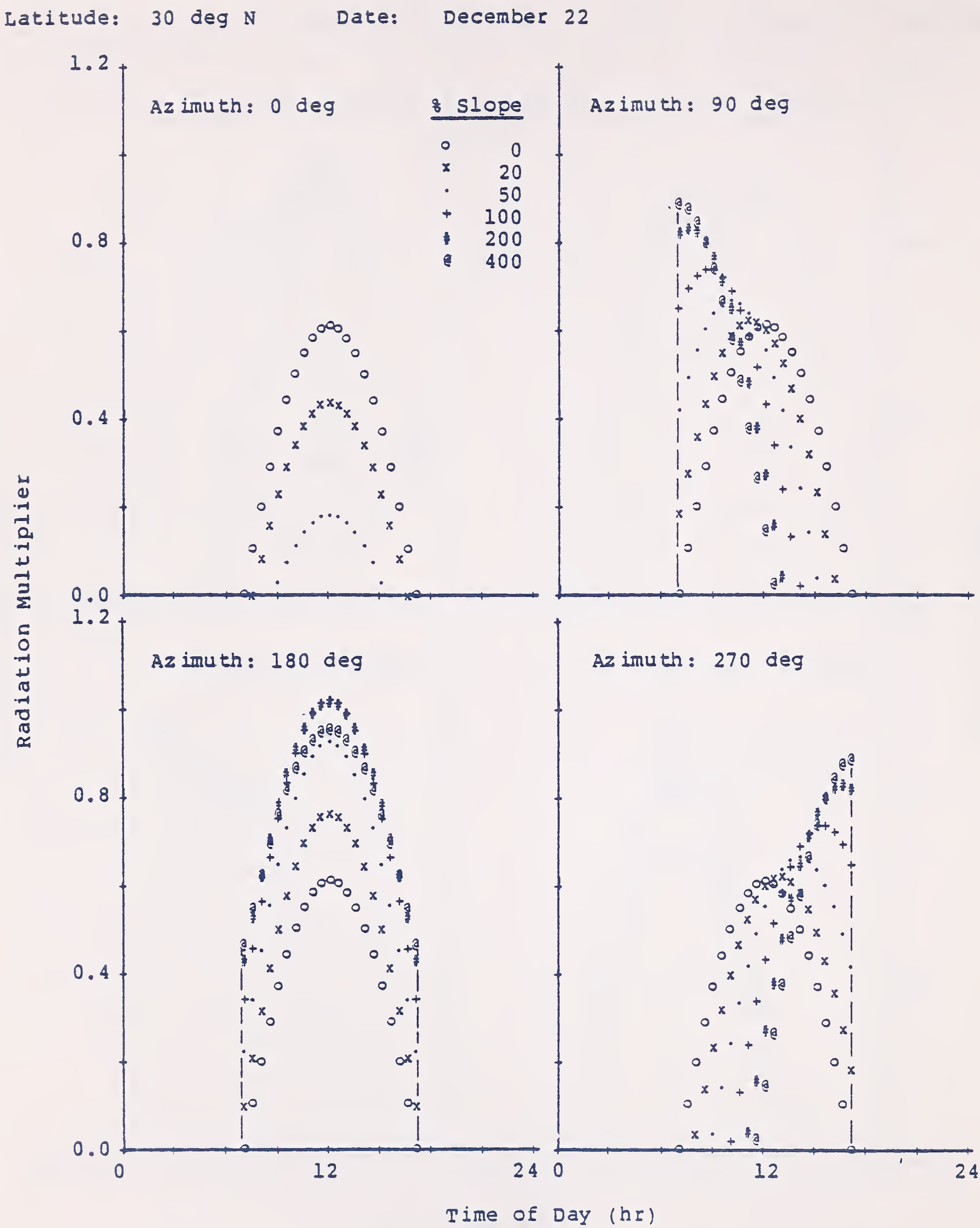

Figure 4.—Instantaneous irradiance multiplier for 30° N latitude on December 22.

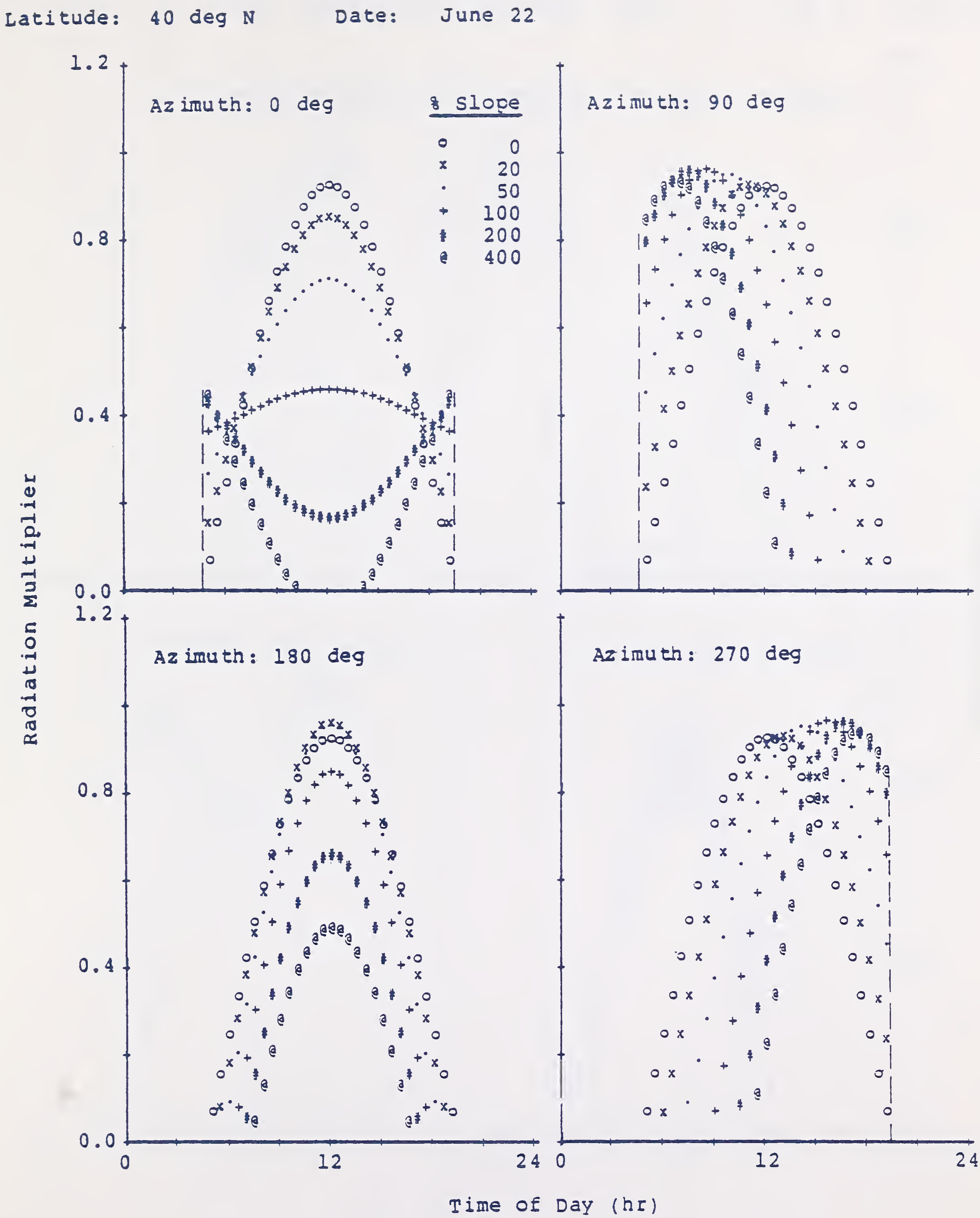

Figure 5.—Instantaneous irradiance multiplier for 40° N latitude on June 22.

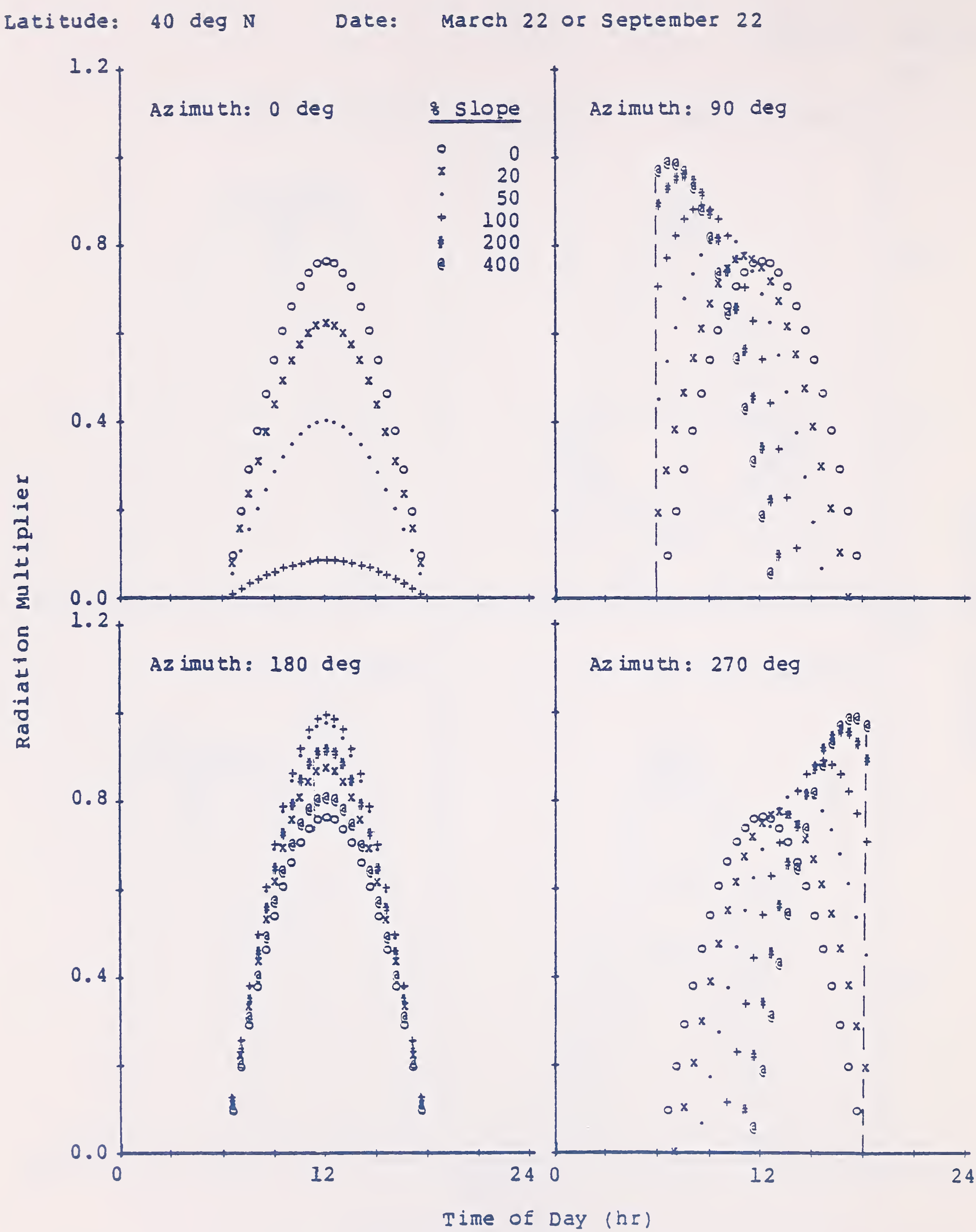

Figure 6.—Instantaneous irradiance multiplier for 40° N latitude on March 22 and September 22.

Latitude: 40 deg N   Date: December 22

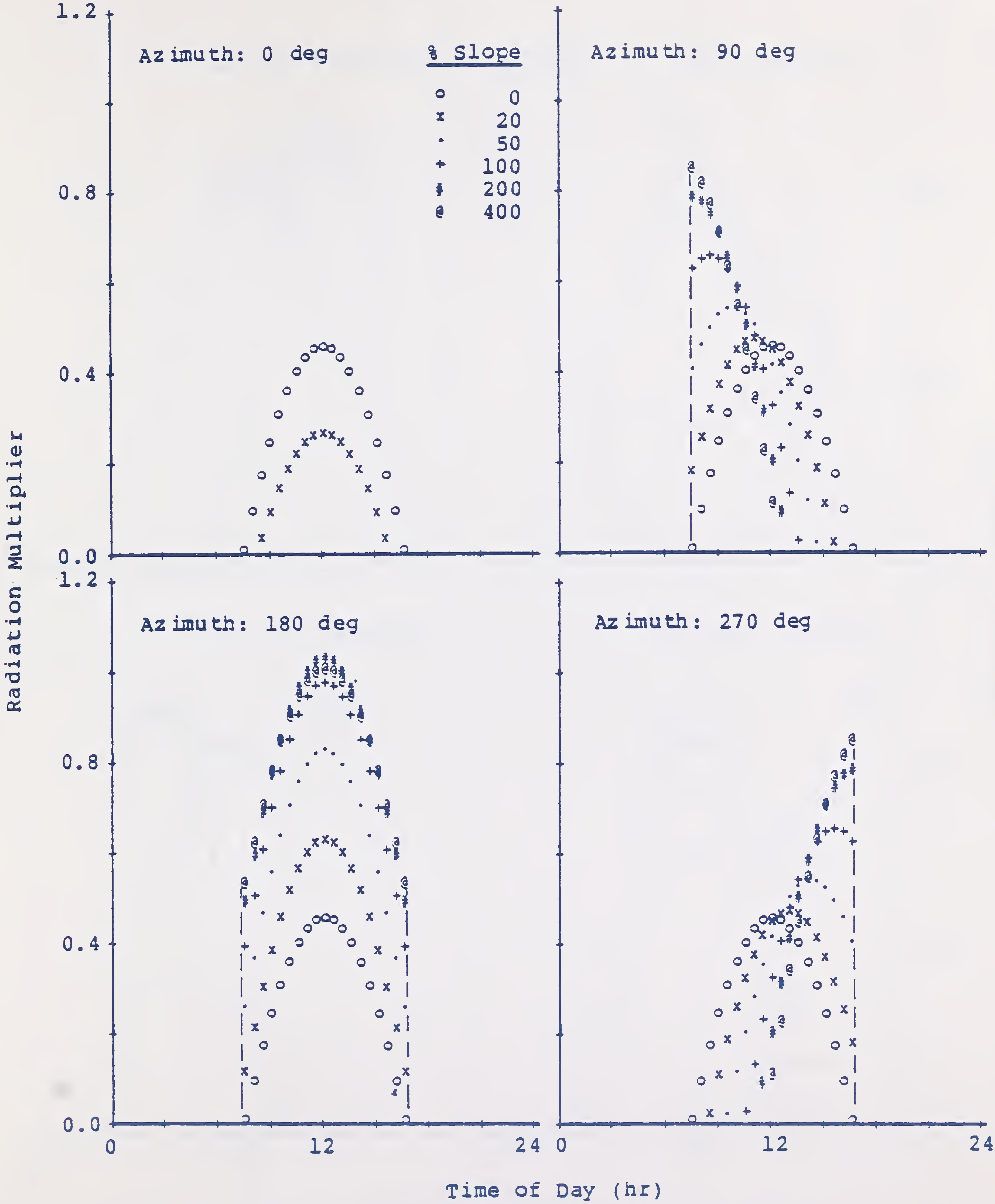

Figure 7.—Instantaneous irradiance multiplier for 40° N latitude on December 22.

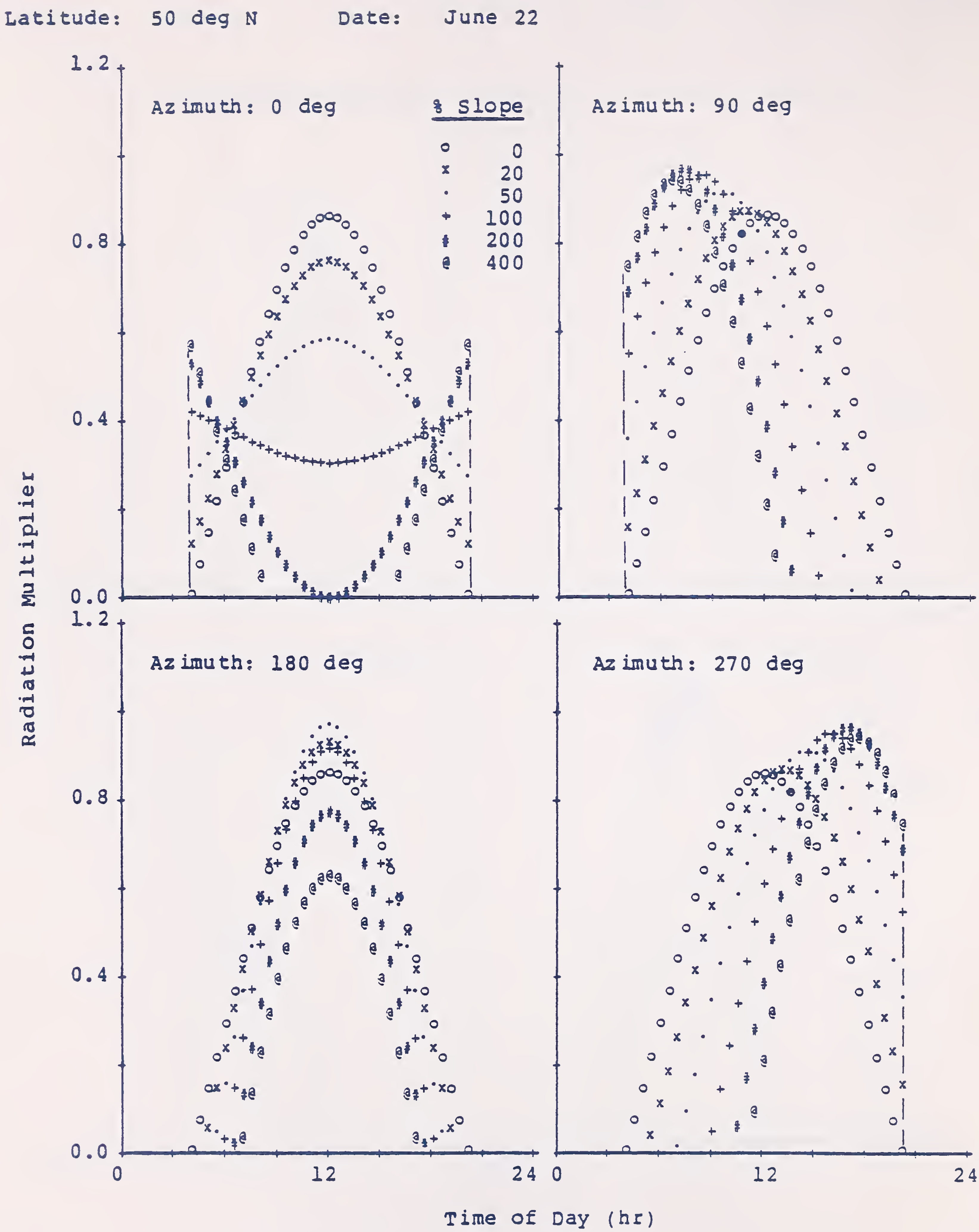

Figure 8.—Instantaneous irradiance multiplier for 50° N latitude on June 22.

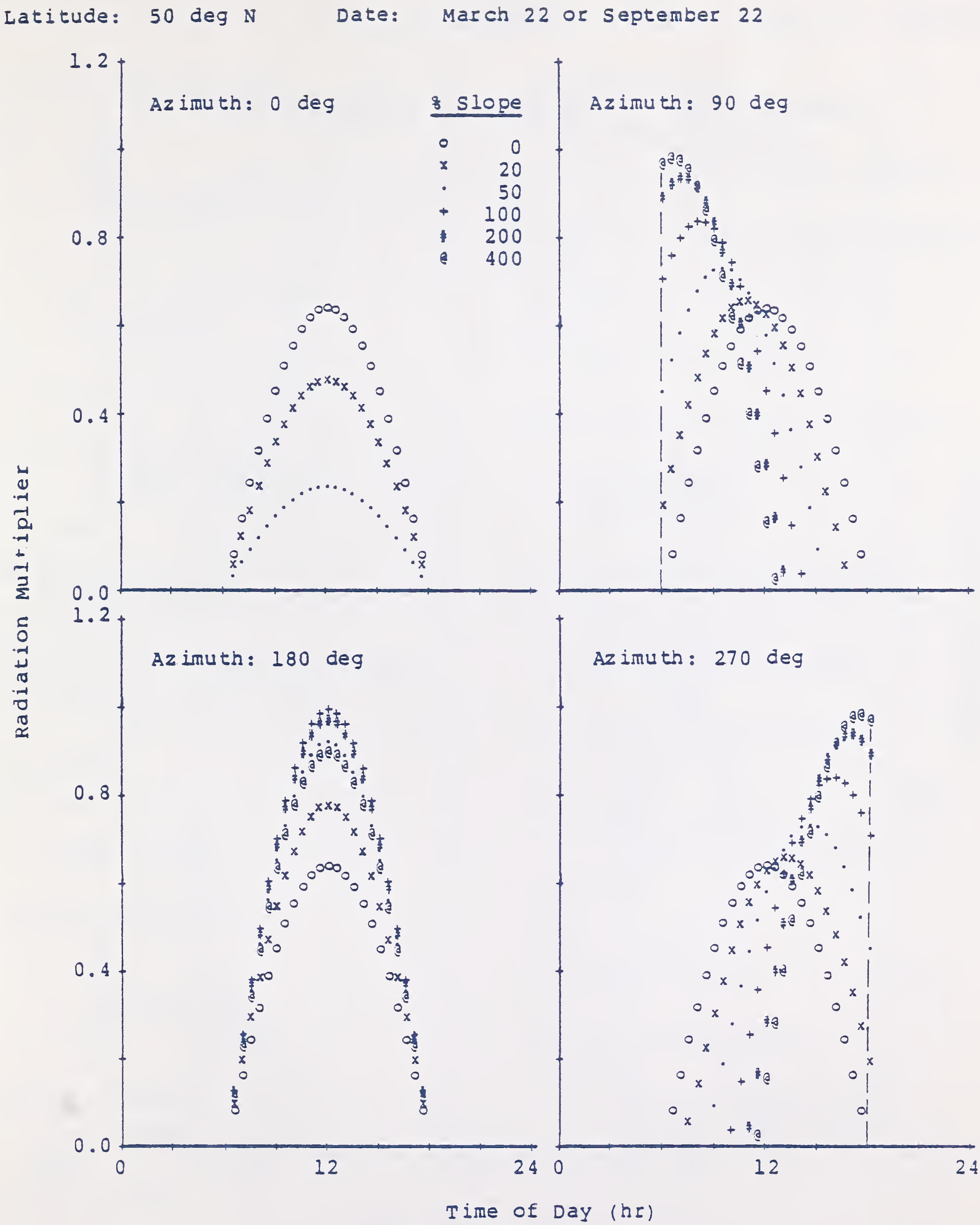

Figure 9.—Instantaneous irradiance multiplier for 50° N latitude on March 22 or September 22.

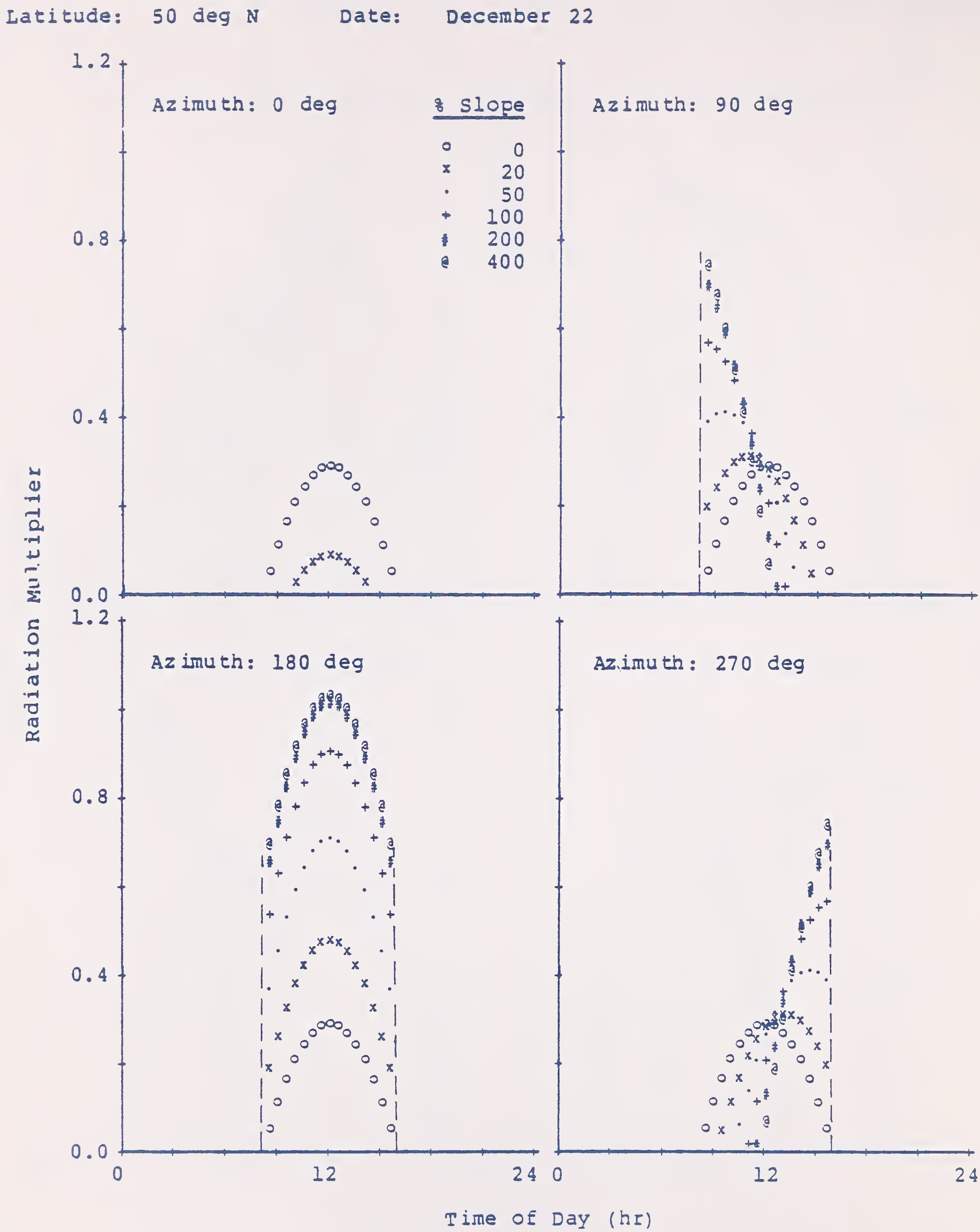

Figure 10.—Instantaneous irradiance multiplier for 50° N latitude on December 22.

Example 1.—Horizontal Surface

Date	June 22
θ	40°
δ	23.5°
R_o	1360 W · m^{-2} (1.95 cal · cm^{-2} · min^{-1})
t	−2.00 hr (1000 hr)

Times of Sunrise and Sunset:

$$\omega t = \cos^{-1}(-\tan 40° \cdot \tan 23.5°)$$
$$t_2 = 7.43 \text{ hr (1926 hr)}$$
$$t_1 = -7.43 \text{ hr (0434 hr)}$$

Instantaneous Irradiance Multiplier:

$$\omega t = -2 \cdot 15° = -30°$$
$$M_i = 1.0[\sin 40° \cdot \sin 23.5° + \cos 40° \cdot \cos 23.5° \cdot \cos(-30°)] \div (0.999847 + 0.001406 \cdot 23.5°)$$

$$M_i = 0.8372$$

Instantaneous Irradiance:

$$R_i = 0.8372 \cdot 1360 \text{ W} \cdot \text{m}^{-2}$$
$$R_i = 1138.6 \text{ W} \cdot \text{m}^{-2}$$

Total Daily Irradiance Multiplier:

$$M_i = 1.0[2 \cdot 7.43 \cdot \sin 40° \cdot \sin 23.5° + (12/\pi) \cdot \cos 40° \cdot \cos 23.5° \cdot 2 \cdot \sin 111.4°] \div 1.0329$$
$$M_t = 8.5251$$

Total Daily Irradiance:

$$R_t = 8.5251 \cdot 60 \cdot 1360$$
$$R_t = 6.956 \times 10^5 \text{ W} \cdot \text{m}^{-2}$$

Example 2.—Tilted Surface

Date	September 22
θ	40°
δ	0°
R_o	1360 W · m^{-2}
t	1.00 hr (1300 hr)
a	90°
i	40%

Inclination in Degrees:

$$i = \tan^{-1}(40/100)$$
$$i = 21.8°$$

Equivalent Latitude:

$$\theta' = \sin^{-1}(\sin 21.8° \cdot \cos 90° \cdot \cos 40° + \cos 21.8° \cdot \sin 40°)$$
$$\theta' = 36.64°$$

Hour Angle Correction:

$$\alpha = \tan^{-1}[(\sin 90° \cdot \sin 21.8°) \div (\cos 21.8° \cdot \cos 40° - \cos 90° \cdot \sin 21.8° \cdot \sin 40°)]$$
$$\alpha = 27.57°$$

Sunrise and Sunset Hour Angles on Equivalent Slope:

$$\omega t' = \cos^{-1}(-\tan 36.64° \cdot \tan 0°)$$
$$\omega t' = 90°$$

Also,

$$\omega t = \cos^{-1}(-\tan 40° \cdot \tan 0°)$$
$$\omega t = 90°$$

Sunrise and Sunset for the Given Slope:

$$t_1 = [\max(-90°, -117.57°)] / 15$$
$$t_1 = -6.00 \text{ hr (0600 hr)}$$
$$t_2 = [\min(90°, 62.43°)] / 15$$
$$t_2 = 4.16 \text{ hr (1610 hr)}$$

Sunrise and Sunset for the Equivalent Slope:

$$t_1' = [\max(-62.43°, -90°)] / 15$$
$$t_1' = -4.16 \text{ hr (0750 hr)}$$
$$t_2' = [\min(117.57°, 90°)] / 15$$
$$t_2' = 6.00 \text{ hr (1800 hr)}$$

Instantaneous Irradiance Multiplier:

$$\omega t'' = 15 \cdot 1.00 + 27.57°$$
$$\omega t'' = 42.57°$$
$$M_i = 1.0[(\sin 36.64° \cdot \sin 0°) + (\cos 36.64° \cdot \cos 0° \cdot \cos 42.57°)] \div (0.999847 + 0.001406 \cdot 0°)$$
$$M_i = 0.5910$$

Instantaneous Irradiance:

$$R_i = 0.5910 \cdot 1360 \text{ W} \cdot \text{m}^{-2}$$
$$R_i = 803.8 \text{ W} \cdot \text{m}^{-2}$$

Total Daily Irradiance Multiplier:

$$M_t = 1.0\{(6.00 + 4.16) \cdot \sin 36.64° \cdot \sin 0° + (12/\pi) \cdot \cos 36.64° \cdot \cos 0° \cdot [\sin 90° - \sin(-62.43°)]\} \div (0.999847 + 0.001406 \cdot 0°)$$
$$M_t = 5.7827$$

Total Daily Irradiance:

$$R_t = 5.7827 \cdot 60 \cdot 1360 \text{ W} \cdot \text{m}^{-2}$$
$$R_t = 4.719 \times 10^5 \text{ W} \cdot \text{m}^{-2}$$

Example 3.—Double Sunrise and Sunset

Date	June 22
θ	50°
δ	23.5°
a	0°
i	250%

Inclination in Degrees:

$i = \tan^{-1}(250/100)$
$i = 68.2°$

Equivalent Latitude:
$\theta' = \sin^{-1}(\sin 68.2° \cdot \cos 0° \cdot \cos 50° + \cos 68.2° \cdot \sin 50°)$
$\theta' = 61.8°$ (note: this latitude is on the opposite side of the North Pole from the given slope; see calculation for α below)

Hour Angle Correction:

$\theta + i = 50 + 68.2$
$\theta + i = 118.2$
Because $(\theta + i) > 90°$, $\alpha = 180°$
$\omega t = \cos^{-1}(-\tan 50° \cdot \tan 23.5°)$
$\omega t = 121.21°$
$\omega t' = \cos^{-1}(-\tan 61.8° \cdot \tan 23.5°)$
$\omega t' = 144.19°$

Times of Double Sunrise and Sunset at the Given Slope:

first $t_1 = -121.21°/15$
first $t_1 = -8.08$ hr (0355 hr)
first $t_2 = (144.19° - 180°) / 15$
first $t_2 = -2.39$ hr (0937 hr)

second $t_1 = (360° - 144.19° - 180°) / 15$
second $t_1 = 2.39$ hr (1423 hr)
second $t_2 = 121.21°/15$
second $t_2 = 8.08$ hr (2005 hr)

Times of Double Sunrise and Sunset at the Equivalent Slope:

first $t_1' = -144.19°/15$
first $t_1' = -9.61$ hr (0223 hr)
first $t_2' = (121.21° + 180° - 360°) / 15$
first $t_2' = -3.92$ hr (0805 hr)

second $t_1' = (-121.21° + 180°) / 15$
second $t_1' = 3.92$ hr (1555 hr)
second $t_2' = 144.19°/15$
second $t_2' = 9.61$ hr (2137 hr)

Appendix B

```
C   THIS PROGRAM ILLUSTRATES THE USAGE OF SUBPROGRAMS
C
C        JULDAT
C        DEC
C        RADVEC
C        RAD
C        GT
C        GI
C        SOLALT
C        SOLAZ
C
C   BY CALCULATING A TOTAL IRRADIATION MULTIPLIER
C   AND A TABLE OF INSTANTANEOUS IRRADIATION
C   MULTIPLIERS, SOLAR ALTITUDES AND SOLAR AZIMUTHS
C   AT 1/4 HOUR INTERVALS FOR 24 HOURS BEGINNING
C   AT SOLAR MIDNIGHT (12 HOURS PAST SOLAR NOON).
C
C   REQUIRED INPUT:
C
C        LATITUDE OF THE SITE IN DEGREES
C        AZIMUTH OF THE SITE MEASURED IN DEGREES
C           CLOCKWISE FROM NORTH
C        SLOPE OF THE SITE IN %
C        INTEGER VALUE OF THE MONTH OF THE PREDICTION
C        INTEGER VALUE OF THE DAY OF MONTH OF THE
C           PREDICTION
C
C   ALL OUTPUT TIMES ARE IN HOURS BEFORE (LT 0) OR AFTER (GT 0)
C   SOLAR NOON.  ALL OUTPUT HOUR ANGLES ARE IN DEGREES
C   CLOCKWISE FROM THE UPPER MERIDIAN.  SOLAR AZIMUTHS ARE GIVEN
C   IN DEGREES CLOCKWISE FROM SOUTH.
C
C
      COMMON/PIE/PI,PI2,PI12,PI180
      DIMENSION GTIME(4),ETIME(4)
      INTEGER DAY
      REAL LAT
      LOGICAL SHADED,DOUBLE
      REAL IRRAD
C
C                                     SET VARIOUS MULTIPLES OF PI FOR
C                                     COMMON BLOCK PIE
C
      PI=4.*ATAN(1.)
      PI2=PI/2.
      PI12=PI/12.
      PI180=PI/180.
C
C                                     READ AND PRINT USER INPUT
C
   10 PRINT 100
  100 FORMAT(1H0,'ENTER LATITUDE IN DEGREES')
      READ(5,*,END=250) LAT
      PRINT 110
  110 FORMAT(1H0,'ENTER AZIMUTH AS DEGREES CLOCKWISE FROM NORTH')
      READ(5,*) AZMUTH
      PRINT 120
  120 FORMAT(1H0,'ENTER % SLOPE')
      READ(5,*) SLOPE
      PRINT 130
  130 FORMAT(1H0,'ENTER MONTH OF PREDICTION')
      READ(5,*) MONTH
      PRINT 140
  140 FORMAT(1H0,'ENTER DAY OF PREDICTION')
      READ(5,*) DAY
C
      PRINT 150, LAT,AZMUTH,SLOPE,MONTH,DAY
  150 FORMAT(1H0/1H ,'LATITUDE - ',F6.1,' DEGREES'/1H ,
     &               'AZIMUTH - ',F6.1,' DEGREES'/1H ,
     &               'SLOPE - ',F6.1,' %'/1H ,
     &               'DATE OF PREDICTION - ',I2,'/',I2)
C
C                                     SET   N - JULIAN DATE
C                                           D - SOLAR DECLINATION
C                                           R2- RADIUS VECTOR SQUARED
C
      N=JULDAT(MONTH,DAY)
      D=DEC(N)
      R2=RADVEC(D)
C                                     DIVISION BY PI180 CONVERTS
C                                     RADIANS TO DEGREES
      PRINT 160, N,D/PI180,SQRT(R2)
  160 FORMAT(1H0,'JULIAN DATE - ',I4/1H ,
     &           'DECLINATION - ',F6.1,' DEGREES'/1H ,
     &           'RADIUS VECTOR - ',F8.5)
C
C                                     CONVERT LATITUDE,AZIMUTH, AND
C                                     SLOPE TO RADIAN MEASURE
C
      RLAT=LAT*PI180
      A=AZMUTH*PI180
      SLP=ATAN(SLOPE/100.)
C
C                                     CALL TO RAD ROUTINE TO CALCULATE
C                                       THETA - EQUIVALENT LATITUDE
C                                       ALPHA - CHANGE IN LONGITUDE
C                                       GTIME - SUNRISE SUNSET AT GIVEN
C                                               SLOPE
C                                       ETIME - SUNRISE SUNSET AT THE
C                                               EQUIVALENT SLOPE
C                                     AND DETERMINE THE NUMBER OF
C                                     SUNRISE SUNSET PERIODS
C
      CALL RAD(RLAT,A,SLP,D,THETA,ALPHA,SHADED,DOUBLE,GTIME,ETIME)
C
      PRINT 170, THETA/PI180,ALPHA/PI180
  170 FORMAT(1H0,'LATITUDE OF EQUIVALENT SLOPE - ',F6.1,' DEGREES'/1H ,
     &           'CHANGE IN LONGITUDE - ',F7.1,' DEGREES')
C
C                                     CALCULATE RADIATION MULTIPLIERS
```

```
C                                  AND GENERATE TABLE
C
      IF(SHADED)  THEN
C                                  GO TO NEXT PROBLEM SINCE
C                                  SUN DOES NOT RISE
         PRINT 180
  180    FORMAT(1H0/1H ,'SLOPE IS SHADED FOR ENTIRE DAY')
      ELSE
C                                  AT LEAST 1 SUNRISE SUNSET PERIOD
         IF(.NOT. DOUBLE)  THEN
C                                  ONE SUNRISE SUNSET PERIOD
C
C                                  DIVISION BY PI12 CONVERTS
C                                  HOUR ANGLES IN RADIANS TO
C                                  HOURS BEFORE OR AFTER SOLAR
C                                  NOON
            PRINT 190, (GTIME(I)/PI12,I=1,2),(ETIME(I)/PI12,I=1,2)
  190       FORMAT(1H0/1H ,'GIVEN SLOPE SUNRISE',F6.1/1H ,
     &                 12X,       'SUNSET',F6.1/1H0,
     &                 'EQUIVALENT SLOPE SUNRISE',F6.1/1H ,
     &                 17X,           'SUNSET',F6.1)
         ELSE
C                                  DOUBLE DAY - IE TWO SUNRISE
C                                  SUNSET PERIODS
            PRINT 200, (GTIME(I)/PI12,I=1,4),(ETIME(I)/PI12,I=1,4)
  200       FORMAT(1H0/1H ,'DOUBLE SUNRISE SUNSET'/1H0,
     &         'GIVEN SLOPE FIRST SUNRISE',F6.1/1H ,
     &         12X,        'FIRST SUNSET',F6.1/1H ,
     &         12X,        'SECOND SUNRISE',F6.1/1H ,
     &         12X,        'SECOND SUNSET',F6.1/1H0,
     &         'EQUIVALENT SLOPE FIRST SUNRISE',F6.1/1H ,
     &         17X,             'FIRST SUNSET',F6.1/1H ,
     &         17X,             'SECOND SUNRISE',F6.1/1H ,
     &         17X,             'SECOND SUNSET',F6.1)
         END IF
C                                  SET TRAD -TOTAL DAILY IRRADIATION
C                                            MULTIPLIER FOR FIRST
C                                            SUNRISE-SET PERIOD
         TRAD=QT(ETIME(1),ETIME(2),THETA,D,R2)
C                                  CALCULATE TRAD FOR SECOND
C                                  SUNRISE-SET PERIOD
         IF(DOUBLE)  TRAD=TRAD+QT(ETIME(3),ETIME(4),THETA,D,R2)
C
         PRINT 210, TRAD
  210    FORMAT(1H0/1H ,'MULTIPLIER FOR TOTAL IRRADIATION - ',F6.2)
         PRINT 220
  220    FORMAT(1H0/1H ,36X,'INSTANTANEOUS'/1H ,
     &      10X,'HOUR',5X,'SOLAR',4X,'SOLAR',4X,
     &      'IRRADIATION'/1H ,
     &      'TIME',5X,'ANGLE',3X,'ALTITUDE',2X,
     &      'AZIMUTH',4X,'MULTIPLIER'/)
C
C                                  CALCULATE TABLE OF SOLAR
C                                  ALTITUDES AND AZIMUTHS FOR
C                                  15 MINUTE INTERVALS
C
            DO 240 I=1,96
C                                  CALCULATE WT - HOUR ANGLE-RADIANS
            WT=(I-48) * (PI12/4.)
C                                  GET  ALT - SOLAR ALTITUDE
C                                       SAZ - SOLAR AZIMUTH
            ALT=SOLALT(WT,RLAT,D)
            SAZ=SOLAZ(WT,RLAT,D,ALT)
C                                  SET IRRAD-INSTANTANEOUS RADIATION
C                                            MULTIPLIER
            IF(.NOT. DOUBLE  .OR.  WT .LE. GTIME(2))  THEN
C                                  EITHER 1 SUNRISE-GTIME(1) AND 1
C                                  SUNSET-GTIME(2) OR DOUBLE SUNRISE
C                                  SUNSET AND WT LE GTIME(2) SO IN
C                                  THE FIRST PERIOD  GTIME(1) TO
C                                  GTIME(2)
               IRRAD=GI(WT,GTIME(1),GTIME(2),THETA,ALPHA,D,R2)
            ELSE
C                                  DOUBLE DAY AND WT GT GTIME(2) SO
C                                  IN SECOND PERIOD GTIME(3) TO
C                                  GTIME(4)
               IRRAD=GI(WT,GTIME(3),GTIME(4),THETA,ALPHA,D,R2)
            END IF
C                                  CALCULATE MILITARY TIME OF DAY
            ITIME=(I/4)*40 + I*15
C                                  PRINT ONE LINE OF TABLE
            PRINT 230, ITIME,WT/PI180,ALT/PI180,SAZ/PI180,IRRAD
  230       FORMAT(1H ,J4,4F10.2)
  240    CONTINUE
      END IF
C                                  GO TO NEXT PROBLEM
      GO TO 10
C
C                                  END OF PROGRAM EXECUTION
C
  250 STOP
      END
```

```
C     SUBROUTINE RAD(TH,A,SLP,C,THP,ALPHA,SHADED,DOUBLE,GTIME,ETIME)
C
C     PURPOSE :
C          GIVEN  TH    - LATITUDE OF A GIVEN SLOPE IN RADIANS
C                          (TH<0 FOR SOUTHERN HEMISPHERE  TH>0 FOR NORTHERN
C                          HEMISPHERE)
C                 A     - AZIMUTH CLOCKWISE FROM NORTH IN RADIANS OF THE GIVEN
C                          SLOPE (0 LE A LT 360)
C                 SLP   - ANGLE OF INCLINATION OF THE GIVEN SLOPE IN RADIANS
C                          (0 LE SLP LE 90)
C                 D    - SOLAR DECLINATION IN RADIANS FOR THE PREDICTION DAY
C                          (-23.45 LE D LE 23.45)
```

```
C
C      THIS ROUTINE WILL CALCULATE
C
C          THP    - LATITUDE OF THE EQUIVALENT SLOPE IN RADIANS
C                   (THP LT 0 FOR SOUTHERN HEMISPHERE  THP GT 0 FOR
C                   NORTHERN HEMISPHERE)
C          ALPHA - CHANGE IN LONGITUDE FROM THE GIVEN SLOPE TO THE
C                   EQUIVALENT SLOPE IN RADIANS (-180 LT ALPHA LE 180)
C          SHADED - IF SHADED=.TRUE. THE GIVEN SLOPE IS SHADED
C                   FROM THE SUN FOR THE ENTIRE DAY
C          DOUBLE - IF DOUBLE=.TRUE. THE GIVEN SLOPE EXPERIENCES
C                   A DOUBLE SUNRISE-SUNSET.  IF DOUBLE=.FALSE. AND
C                   SHADED=.FALSE. THE GIVEN SLOPE EXPERIENCES ONE
C                   SUNRISE AND ONE SUNSET
C          GTIME(1) - HOUR ANGLE OF THE FIRST SUNRISE AT THE
C                   GIVEN SLOPE IN RADIANS (-180 LE GTIME(1) LE 180)
C          GTIME(2) - HOUR ANGLE OF THE FIRST SUNSET AT THE
C                   GIVEN SLOPE IN RADIANS (-180 LE GTIME(2) LE 180)
C          GTIME(3) - HOUR ANGLE OF THE SECOND SUNRISE AT THE
C                   GIVEN SLOPE IN RADIANS (-180 LE GTIME(3) LE 180)
C          GTIME(4) - HOUR ANGLE OF THE SECOND SUNSET AT THE
C                   GIVEN SLOPE IN RADIANS (-180 LE GTIME(4) LE 180)
C          ETIME(1) - HOUR ANGLE OF THE FIRST SUNRISE AT THE
C                   EQUIVALENT SLOPE IN RADIANS (-180 LE ETIME(1) LE 180)
C          ETIME(2) - HOUR ANGLE OF THE FIRST SUNSET AT THE
C                   EQUIVALENT SLOPE IN RADIANS (-180 LE ETIME(2) LE 180)
C          ETIME(3) - HOUR ANGLE OF THE SECOND SUNRISE AT THE
C                   EQUIVALENT SLOPE IN RADIANS (-180 LE ETIME(3) LE 180)
C          ETIME(4) - HOUR ANGLE OF THE SECOND SUNSET AT THE
C                   EQUIVALENT SLOPE IN RADIANS (-180 LE ETIME(4) LE 180)
C
C      NOTE THAT IF SHADED=.TRUE. GTIME AND ETIME ARE MEANINGLESS.
C      IF SHADED=.FALSE. AND .DOUBLE=.FALSE. THEN GTIME(3), GTIME(4),
C      ETIME(3), AND ETIME(4) ARE MEANINGLESS.
C
C      ONCE THESE VARIABLES ARE COMPUTED THE USER MAY INCORPORATE
C      SUBPROGRAMS GI AND GT TO CALCULATE IRRADIATION MULTIPLIERS.
C
C  REQUIRED SUBPROGRAMS : NONE
C
C  PROGRAMMING NOTES :
C
C      USAGE - CALL RAD(TH,A,SLP,D,THP,ALPHA,SHADED,DOUBLE,GTIME,ETIME)
C
C      PARAMETER TYPES :
C
C           PARAMETERS TH,A,SLP,D,THP,ALPHA ARE SCALARS OF TYPE REAL
C           PARAMETERS SHADED,DOUBLE ARE SCALARS OF TYPE LOGICAL
C           PARAMETERS GTIME,ETIME ARE ONE DIMENSIONAL ARRAYS OF
C               TYPE REAL AND DIMENSION 4
C           ALL VARIABLES ARE SINGLE PRECISION
C
C      COMMON BLOCKS - COMMON/PIE/PI,PI2,PI12,PI180    WHERE
C           PI=4*ATAN(1)
C           PI2=PI/2
C           PI12=PI/12
C                PI180=PI/180
C
C
      SUBROUTINE RAD(TH,A,SLP,D,THP,ALPHA,SHADED,DOUBLE,GTIME,ETIME)
      COMMON/PIE/PI,PI2,PI12,PI180
      REAL TH,A,SLP,THP,ALPHA,GTIME(4),ETIME(4)
      LOGICAL SHADED,DOUBLE

CALCULATE LAT OF EQUIV SLOPE - THP

      THP=SIN(SLP)*COS(A)*COS(TH)  +  COS(SLP)*SIN(TH)
      THP=ASIN(THP)

CALCULATE CHANGE IN LONGITUDE - ALPHA

      IF(A .GT. 0.)  THEN
        SINA=SIN(A)
        ALPHA=COS(SLP)*COS(TH)  -  COS(A)*SIN(SLP)*SIN(TH)
        ALPHA=(SINA*SIN(SLP))/ALPHA
        ALPHA=ATAN(ALPHA)
        IF(SINA*ALPHA .LT. 0.)  THEN
           ALPHA=SIGN(PI,SINA)+ALPHA
        END IF
      ELSE
        IF((ABS(TH)+SLP) .GT. PI2)  THEN
          ALPHA=PI
        ELSE
          ALPHA=0.
        END IF
      END IF

CALCULATE    WT - HOUR ANGLE OF SUNSET AT GIVEN SLOPE
C             WTP - HOUR ANGLE OF SUNSET AT EQUIVALENT SLOPE

      IF(ABS(TH)+ABS(D) .GT. PI2)  THEN
        IF(TH*D .LT. 0.)  THEN
          WT=0.
        ELSE
          WT=PI
        END IF
      ELSE
        WT=ACOS(-TAN(TH)*TAN(D))
      END IF
      IF(ABS(THP)+ABS(D) .GT. PI2)  THEN
        IF(THP*D .LT. 0.)  THEN
              WTP=0.
            ELSE
              WTP=PI
            END IF
          ELSE
            WTP=ACOS(-TAN(THP)*TAN(D))
          END IF

      CALCULATE SUNRISE SUNSET HOUR ANGLES

          T1=-WTP
```

```
      T2=(WT+ALPHA)-(2.*PI)
      T3=-WT+ALPHA
      T4=WTP

      IF(T4 .LE. T3)  THEN
         SHADED=.TRUE.
         DOUBLE=.FALSE.
      ELSE
         SHADED=.FALSE.
         IF(T1 .LT. T2 .AND. T2 .LT. T3  .AND.  T3 .LT. T4)  THEN
            DOUBLE=.TRUE.
            GTIME(1)=-WT
            GTIME(2)=WTP-ALPHA
            GTIME(3)=(2.*PI) - (WTP+ALPHA)
            GTIME(4)=WT
            ETIME(1)=T1
            ETIME(2)=T2
            ETIME(3)=T3
            ETIME(4)=T4
         ELSE
            DOUBLE=.FALSE.
            GTIME(1)=AMAX1(-WT,-WTP-ALPHA)
            GTIME(2)=AMIN1(WT,WTP-ALPHA)
            ETIME(1)=AMAX1(-WT+ALPHA,-WTP)
            ETIME(2)=AMIN1(WT+ALPHA,WTP)
         END IF
      END IF
      RETURN
      END
```

```
C     REAL FUNCTION DEC(N)
C
C     PURPOSE :
C          GIVEN  N      -THE JULIAN DATE
C          THIS ROUTINE WILL CALCULATE THE CORRESPONDING SOLAR DECLINATION
C                 IN RADIANS   (-23.45 LE DEC LE 23.45)
C
C     REQUIRED SUBPROGRAMS : NONE
C
C
C     PROGRAMMING NOTES :
C
C          USAGE - D=DEC(N)
C
C          PARAMETER TYPES :
C
C               PARAMETER N IS A SCALAR OF TYPE INTEGER
C
C          COMMON BLOCKS - COMMON/PIE/PI,PI2,PI12,PI180    WHERE
C               PI=4*ATAN(1)
C               PI2=PI/2
C               PI12=PI/12
C               PI180=PI/180
C
C
C
      REAL FUNCTION DEC(N)
      COMMON/PIE/PI,PI2,PI12,PI180
      INTEGER N
C
      DEC=23.45*PI180*SIN(2.*PI*(284+N)/365.)
C
      RETURN
      END
```

```
C     INTEGER FUNCTION  JULDAT(MONTH,DAY)
C
C     PURPOSE :
C          GIVEN  MONTH - MONTH FOR PREDICTION (1 LE MONTH LE 12)
C                 DAY   - DAY OF MONTH FOR PREDICTION (1 LE DAY LE 31)
C          THIS ROUTINE WILL CALCULATE THE CORRESPONDING JULIAN DATE.
C
C     REQUIRED SUBPROGRAMS : NONE
C
C     PROGRAMMING NOTES :
C
C          USAGE - N=JULDAT(MONTH,DAY)
C
C          PARAMETER TYPES :
C
C               PARAMETERS MONTH,DAY ARE SCALARS OF TYPE INTEGER
C
C          COMMON BLOCKS : NONE
C
C
      INTEGER FUNCTION JULDAT(MONTH,DAY)
      INTEGER MONTH,DAY
C
      N=31*(MONTH-1) + DAY
      IF(MONTH .GT. 2)  THEN
         JULDAT=N + NINT(-.4*MONTH - 1.8)
      ELSE
         JULDAT=N
      END IF
      RETURN
      END
```

```
C     REAL FUNCTION QI(TIME,SR,SS,THETA,ALPHA,DEC,R2)
C
C     PURPOSE :
C          GIVEN  TIME   - HOUR ANGLE IN RADIANS FROM SOLAR NOON
C                          AT THE GIVEN SLOPE (-180 LE TIME LE 180)
C                 SR     - HOUR ANGLE IN RADIANS OF SUNRISE AT THE
C                          GIVEN SLOPE (-180 LE SR LE 180)
C                 SS     - HOUR ANGLE IN RADIANS OF SUNSET AT THE
C                          GIVEN SLOPE (-180 LE SS LE 180)
C                 THETA - LATITUDE OF THE EQUIVALENT SLOPE IN RADIANS
C                          (-90 LE THETA LE 90)
C                 ALPHA - CHANGE IN LONGITUDE FROM THE GIVEN SLOPE TO THE
C                          EQUIVALENT SLOPE IN RADIANS (-180 LT ALPHA LE 180)
C                 DEC   - SOLAR DECLINATION IN RADIANS FOR A PARTICULAR DAY
C                          (-23.45 LE DEC LE 23.45)
C                 R2    - THE SQUARED RATIO OF THE EARTH-SUN DISTANCE
C                          TO ITS MEAN FOR A PARTICULAR DAY
C          THIS ROUTINE WILL CALCULATE AN INSTANTANEOUS IRRADIATION
C               MULTIPLIER CORRESPONDING TO HOUR ANGLE TIME.
C
C     REQUIRED SUBPROGRAMS : NONE
C
C     PROGRAMMING NOTES :
C
C          USAGE - IRRAD=QI(TIME,SR,SS,THETA,ALPHA,DEC,R2)
C
C          PARAMETER TYPES :
C
C           PARAMETERS TIME,SR,SS,THETA,ALPHA,DEC,R2 ARE SCALARS OF TYPE REAL
C
C          COMMON BLOCKS : NONE
C
C
        REAL FUNCTION QI(TIME,SR,SS,THETA,ALPHA,DEC,R2)
        REAL TIME,SR,SS,THETA,ALPHA,DEC,R2
C
        IF(TIME .LT. SR  .OR.  TIME .GT. SS)  THEN
           QI=0.
        ELSE
           WTPP=TIME + ALPHA
           TERM1=SIN(THETA)*SIN(DEC)
           TERM2=COS(THETA)*COS(DEC)*COS(WTPP)
           QI=(1.0/R2) * (TERM1 + TERM2)
C                                        THE RANGE OF QI IS MATHEMATICALLY
C                                        LIMITED TO  0 LE QI LE 1.  DUE TO
C                                        ROUNDOFF AND/OR CANCELLATION ERRORS
C                                        THE ABOVE STATEMENT MAY ASSIGN A
C                                        VALUE LESS THAN 0 OR GREATER THAN 1
C                                        TO QI.  THE FOLLOWING STATEMENTS
C                                        ALLEVIATE THE PROBLEM.
           IF(QI .LT. 0.)  QI=0.
           IF(QI .GT. 1.)  QI=1.
        END IF
        RETURN
        END
```

```
C     REAL FUNCTION QT(SR,SS,THETA,DEC,R2)
C
C     PURPOSE :
C          GIVEN  SR     - HOUR ANGLE IN RADIANS OF SUNRISE AT THE
C                          SLOPE (-180 LE SR LE 180)
C                 SS     - HOUR ANGLE IN RADIANS OF SUNSET AT THE
C                          SLOPE (-180 LE SS LE 180)
C                 THETA - LATITUDE OF THE SLOPE IN RADIANS
C                          (-90 LE THETA LE 90)
C                 DEC   - SOLAR DECLINATION IN RADIANS FOR A PARTICULAR DAY
C                          (-23.45 LE DEC LE 23.45)
C                 R2    - THE SQUARED RATIO OF THE EARTH-SUN DISTANCE
C                          TO ITS MEAN FOR A PARTICULAR DAY
C          THIS ROUTINE WILL CALCULATE A TOTAL IRRADIATION MULTIPLIER
C               CORRESPONDING TO HOUR ANGLES SR SS.
C
C     REQUIRED SUBPROGRAMS : NONE
C
C     PROGRAMMING NOTES :
C
C          USAGE - TRAD=QT(SR,SS,THETA,DEC,R2)
C
C          PARAMETER TYPES :
C
C               PARAMETERS SR,SS,THETA,DEC,R2 ARE SCALARS OF TYPE REAL
C
C          COMMON BLOCKS - COMMON/PIE/PI,PI2,PI12,PI180   WHERE
C               PI=4*ATAN(1)
C               PI2=PI/2
C               PI12=PI/12
C               PI180=PI/180
C
C
        REAL FUNCTION QT(SR,SS,THETA,DEC,R2)
        REAL SR,SS,THETA,DEC,R2
        COMMON/PIE/PI,PI2,PI12,PI180
C
        SSMSR=(SS-SR)*12./PI
        TERM1=SSMSR*SIN(THETA)*SIN(DEC)
        TERM2=COS(THETA)*COS(DEC)*(SIN(SS)-SIN(SR))
        QT=(1.0/R2)*(TERM1 + 12./PI*TERM2)
C                                        THE RANGE OF QT IS MATHEMATICALLY
C                                        LIMITED TO :
C
C                                        0   LE   QT   LE   (1/R2)(SSMSR+24/PI)
C
C                                        DUE TO ROUNDING AND/OR CANCELLATION
C                                        ERRORS THE ABOVE STATEMENT MAY ASSIGN
C                                        A VALUE TO QT OUT OF THIS RANGE.  THE
C                                        FOLLOWING STATEMENTS ALLEVIATE THE
C                                        PROBLEM.
        IF(QT .LT. 0.)  QT=0.
        IF(QT .GT. 1./R2*(SSMSR+24./PI)) QT=1./R2*(SSMSR+24./PI)
C
        RETURN
        END
```

```
C     REAL FUNCTION RADVEC(D)
C
C     PURPOSE :
C          GIVEN  D        - THE SOLAR DECLINATION IN RADIANS
C          THIS ROUTINE WILL CALCULATE THE SQUARED RATIO OF
C               THE EARTH-SUN DISTANCE TO ITS MEAN FOR THE
C               CORRESPONDING DAY.
C
C     REQUIRED SUBPROGRAMS : NONE
C
C     PROGRAMMING NOTES :
C
C          USAGE - R2=RADVEC(D)
C
C          PARAMETER TYPES :
C
C               PARAMETER D IS A SCALAR OF TYPE REAL
C
C          COMMON BLOCKS - COMMON/PIE/PI,PI2,PI12,PI180   WHERE
C               PI=4*ATAN(1)
C               PI2=PI/2
C               PI12=PI/12
C               PI180=PI/180
C
C
      REAL FUNCTION RADVEC(D)
      COMMON/PIE/PI,PI2,PI12,PI180
      REAL D
C
      RADVEC= 0.999847 + 0.001406 * (D/PI180)
C
      RETURN
      END
```

```
C     REAL FUNCTION SOLALT(WT,LAT,DEC)
C
C     PURPOSE :
C          GIVEN  WT    - THE HOUR ANGLE IN RADIANS (-180 LE WT LE 180)
C                 LAT   - THE LATITUDE IN RADIANS OF A GIVEN SITE
C                           (-90 LE LAT LE 90)
C                 DEC   - THE SOLAR DECLINATION IN RADIANS FOR A GIVEN DAY
C                           (-23.45 LE DEC LE 23.45)
C          THIS ROUTINE WILL CALCULATE THE SOLAR ALTITUDE IN RADIANS
C               FROM THE TRUE HORIZON.
C
C     REQUIRED SUBPROGRAMS : NONE
C
C     PROGRAMMING NOTES :
C
C          USAGE - ALT=SOLALT(WT,LAT,DEC)
C
C          PARAMETER TYPES :
C
C               PARAMETERS WT,LAT,DEC ARE SCALARS OF TYPE REAL
C
C          COMMON BLOCKS : NONE
C
C
C
      REAL FUNCTION SOLALT(WT,LAT,DEC)
      REAL WT,LAT,DEC
C
      SINALT=COS(WT)*COS(LAT)*COS(DEC) + SIN(LAT)*SIN(DEC)
C
C                             DUE TO ROUNDING AND/OR CANCELLATION
C                             ERRORS THE ABOVE STATEMENT MAY ASSIGN
C                             A VALUE LESS THAN -1 OR GREATER THAN 1
C                             TO SINALT CAUSING AN ERROR TERMINATION
C                             IN THE ASIN ROUTINE.  THE FOLLOWING
C                             STATEMENT ELIMINATES THIS PROBLEM.
C
      SINALT=SIGN(AMIN1(ABS(SINALT),1.),SINALT)
C
      SOLALT=ASIN(SINALT)
C
      RETURN
      END
```

```
C     REAL FUNCTION SOLAZ(WT,LAT,DEC,ALT)
C
C     PURPOSE :
C          GIVEN  WT    - THE HOUR ANGLE IN RADIANS (-180 LE WT LE 180)
C                 LAT   - THE LATITUDE IN RADIANS OF A GIVEN SITE
C                         (-90 LE LAT LE 90)
C                 DEC   - THE SOLAR DECLINATION IN RADIANS FOR A GIVEN DAY
C                          (-23.45 LE DEC LE 23.45)
C                 ALT   - THE SOLAR ALTITUDE IN RADIANS FROM THE
C                          TRUE HORIZON (-90 LE ALT LE 90)
C          THIS ROUTINE WILL CALCULATE THE SOLAR AZIMUTH IN RADIANS
C               MEASURED CLOCKWISE FROM SOUTH (0 LE SOLAZ LT 360).
C
C     REQUIRED SUBPROGRAMS : NONE
C
C     PROGRAMMING NOTES :
C
C          USAGE - AZ=SOLAZ(WT,LAT,DEC,ALT)
C
C          PARAMETER TYPES :
C
C               PARAMETERS WT,LAT,DEC,ALT ARE SCALARS OF TYPE REAL
C
C          COMMON BLOCKS - COMMON/PIE/PI,PI2,PI12,PI180   WHERE
C               PI=4*ATAN(1)
C               PI2=PI/2
C               PI12=PI/12
C               PI180=PI/180
C
C
      REAL FUNCTION SOLAZ(WT,LAT,DEC,ALT)
      REAL WT,LAT,DEC,ALT
      COMMON/PIE/PI,PI2,PI12,PI180

      ABSWT=ABS(WT)
      ABSLAT=ABS(LAT)
      ABSDEC=ABS(DEC)
      IF(ABS(ALT) .LT. PI2)  THEN
         SINSAZ=COS(DEC)*SIN(WT)/COS(ALT)
C
C                                  WHEN WT AND ALT SHOULD DIFFER BY + OR - 90
C                                  DEGREES BUT DONT DUE TO ROUNDOFF AND/OR
C                                  CANCELLATION ERRORS THE ABOVE STATMENT
C                                  MAY ASSIGN A VALUE LESS THAN -1 OR
C                                  GREATER THAN +1 TO SINSAZ CAUSING AN
C                                  ERROR TERMINATION IN THE ASIN ROUTINE.
C                                  THE FOLLOWING STATEMENT ELIMINATES
C                                  THIS PROBLEM.
C
         SINSAZ=SIGN(AMIN1(ABS(SINSAZ),1.),SINSAZ)
C
         SAZ=ASIN(SINSAZ)
      END IF
      IF(ABSLAT .GT. ABSDEC)  THEN
         HR=ACOS(TAN(DEC)*COTAN(LAT))
         IF(ABSWT .LE. HR)  THEN
            SOLAZ=SAZ
         ELSE
            SOLAZ=SIGN(PI,WT) - SAZ
         END IF
      ELSE
         IF(0. .LT. ABSWT  .AND.  ABSWT .LT. PI)  THEN
            IF(DEC .EQ. 0.)  THEN
               SOLAZ=SIGN(PI2,WT)
            ELSE IF(DEC .LT. 0.)  THEN
               SOLAZ=SAZ
            ELSE
               SOLAZ=SIGN(PI,WT)-SAZ
            END IF
         ELSE IF(ABSWT .EQ. PI)  THEN
            IF(ABSLAT .EQ. ABSDEC)  THEN
               IF(LAT .GE. 0)  THEN
                  SOLAZ=PI
               ELSE
                  SOLAZ=0.
               END IF
            ELSE
               IF(DEC .GT. 0.)  THEN
                  SOLAZ=PI
               ELSE
                  SOLAZ=0.
               END IF
            END IF
         ELSE
            IF(ABSLAT .EQ. ABSDEC)  THEN
               IF(LAT .GE. 0.)  THEN
                  SOLAZ=0.
               ELSE
                  SOLAZ=PI
               END IF
            ELSE
               IF(DEC .GT. 0.)  THEN
                  SOLAZ=PI
               ELSE
                  SOLAZ=0.
               END IF
            END IF
         END IF
      END IF
      IF(SOLAZ .LT. 0.)  SOLAZ=2.*PI + SOLAZ
C
      RETURN
      END
```

U.S. Department of Agriculture
Forest Service

Rocky Mountain Forest and Range Experiment Station

Rocky Mountains

Southwest

Great Plains

The Rocky Mountain Station is one of eight regional experiment stations, plus the Forest Products Laboratory and the Washington Office Staff, that make up the Forest Service research organization.

RESEARCH FOCUS

Research programs at the Rocky Mountain Station are coordinated with area universities and with other institutions. Many studies are conducted on a cooperative basis to accelerate solutions to problems involving range, water, wildlife and fish habitat, human and community development, timber, recreation, protection, and multiresource evaluation.

RESEARCH LOCATIONS

Research Work Units of the Rocky Mountain Station are operated in cooperation with universities in the following cities:

Albuquerque, New Mexico
Bottineau, North Dakota
Flagstaff, Arizona
Fort Collins, Colorado*
Laramie, Wyoming
Lincoln, Nebraska
Lubbock, Texas
Rapid City, South Dakota
Tempe, Arizona

*Station Headquarters: 240 W. Prospect St., Fort Collins, CO 80526